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PROPAGATION OF RAYLEIGH WAVES IN DISSIPATIVE MEDIA. LINE SOURCE

S. Z. Dunin and G. A. Maksimov

UDC 550.344.43:550.344.56:550.347.34

Surface acoustic waves and, in particular, Rayleigh waves have attracted the attention of researchers in a number of areas of science and technology, such as seismologists and creators of microelectronic technology. This is due to the specific features of surface waves. Thus, in seismological measurements of a wave field at large distances from the source its most clearly recorded component is often the surface wave, due to the fact that its damping is weaker than that of bulk waves. In microelectronics one uses the fact that surface waves, possessing a low velocity in comparison with the propagation velocity of electromagnetic waves, as well as localization in the near-surface layer, make it possible to create very compact electron devices. In both seismology and microelectronics it is important to know how the wave profile and amplitude change with its propagation in the medium. These variations may be a consequence of the dispersion-dissipative properties of the medium. The necessity of keeping these properties in mind becomes clear if it is taken into account that in seismology one deals with wave propagation at very large distances, while in microelectronics one uses very short pulses. This brings about the possibility of a strong effect of dispersion-dissipative medium properties on the propagation of surface waves.

There exists a number of studies devoted to the study of the effect of nonideal medium properties on propagation of Rayleigh waves [1-4]. For example, a relation was found in [2] between small corrections to the wave vector of a monochromatic Rayleigh wave and similar corrections to the wave vectors of longitudinal and transverse bulk waves. A numerical calculation was carried out in [3] of the profile of a Rayleigh wave, excited by a shock near the surface. The dispersion-dissipative properties of the medium were accounted for by a linear frequency dependence of the imaginary part of the wave vectors of longitudinal and transverse waves. It was noted in [4] that substantial difficulties arise in attempting to account for the effect of dispersion-dissipative properties of the medium on the wave field evolution in half-space by using the mathematical apparatus developed for an elastic medium. A successive method of calculating the wave fields in dispersion-dissipative media was first

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 141-149, May-June, 1988. Original article submitted March 16, 1987.

0021-8944/88/2903-0443$12.50

suggested in [1]. It makes it possible to avoid the difficulties mentioned, and provide the answer to a number of practically interesting questions: under what conditions do the dispersion-dissipative properties of the medium affect substantially the propagation of a Rayleigh wave? How do its profile and amplitude vary in this case? What is the role of parameters of a nonmonochromatic source, such as its depth and radiation time, as well as medium properties, such as dispersion and relaxation time?

Statement of the Problem and Basic Relations. In a homogeneous isotropic half-space, filled by a linear inelastic medium with an equation of state of hereditary type of general form

$$\sigma_{ij}(\mathbf{r}, t) = \int_0^t M(t-t')\,\varepsilon_{ij}(\mathbf{r}, t')\,dt' + \delta_{ij}\int_0^t L(t-t')\,\varepsilon_{kk}(\mathbf{r}, t')\,dt', \tag{1}$$

at a depth h parallel to the surface is located a line source, relating longitudinal waves with a source function $Q_u(t)$. The surface S of the half-space is assumed free, i.e., $\sigma_{ij}n_j/S = 0$ ($\mathbf{n}$ is the normal vector to the surface), while for $t < 0$ the half-space is in rest. Equation (1), along with the equation of motion and the relation between the deformation tensor ε_{ij} and the displacements u_i in the linear approximation $\rho_0\partial^2 u_i/\partial t^2 = \partial\sigma_{ik}/\partial x_k$, $\varepsilon_{ik} = \partial u_i/\partial x_k + \partial u_k/\partial x_i$, form a closed system for determining the displacement field in half-space.

Introduce the scalar A^0 and vector $\mathbf{A}$ potentials $A^0 = \mathbf{u}_1$, $\mathrm{rot}\,\mathbf{A} = \mathbf{u}_2$ ($\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2$) grad. For the Laplace transforms in time $A^0(\mathbf{r}, p)$ and $\mathbf{A}(\mathbf{r}, p)$ we obtain the system

$$\begin{gathered}[\Delta - K_1^2(p)]\,A^0(\mathbf{r}, p) = -Q_u(p)\,\delta(\mathbf{r} - \mathbf{R}(\mathbf{r})),\\ K_1^2(p) = \rho_0 p^2/[2(M(p) + L(p))],\\ [\Delta - K_2^2(p)]\,\mathbf{A}(\mathbf{r}, p) = 0,\quad \sigma_{ij}(A^0, \mathbf{A})\,n_j\,|\,S = 0,\\ A^0(\mathbf{r}\to\infty, p) = \mathbf{A}(\mathbf{r}\to\infty, p) = 0,\quad K_2^2(p) = \rho_0 p^2/M(p).\end{gathered} \tag{2}$$

Here the initial conditions are vanishing, and $\mathbf{R}(\mathbf{r})$ are the coordinates of the line source. Denote by A_0^α the solution of the system with $Q_u(p)$. We note that due to the homogeneity of the boundary conditions in M(p) and L(p) the solution A_0^α depends only on the parameters $K_1(p)$ and $K_2(p)$: $A_0^\alpha(\mathbf{r}, p) = A_0^\alpha(\mathbf{r}, K_1(p), K_2(p))$. In the special case of an elastic wave, for which $K_1(p) = p/c_\ell$, $K_2(p) = p/c_s$ (c_ℓ and c_s are the velocities of longitudinal and shear waves), the solution A_0^α has the form $A_{00}^\alpha = A_0^\alpha(\mathbf{r}, p/c_l, p/c_s)$. The solution of system (2) for $Q_u(p) \neq 1$ differs from A_0^α by the factor $Q_u(p)$: $A^\alpha(\mathbf{r}, p) = Q_u(p)A_0^\alpha(\mathbf{r}, K_1(p), K_2(p))$, $\alpha = 0, 1, 2, 3$. The displacement field can now be found by differentiation with respect to the spatial coordinates $u_i = \partial A^0/\partial x_i + \varepsilon_{ijk}\partial A^k/\partial x_j$, $\mathbf{u}(\mathbf{r}, p) = Q_u(p)\tilde{\mathbf{G}}(\mathbf{r}, K_1(p), K_2(p))$ ($\mathbf{G}$ is the Green's function of the displacement field). The space–time representation of the displacement wave field is given by the Mellin integral

$$\mathbf{u}(\mathbf{r}, t) = \frac{1}{2\pi i}\int_\gamma dp\,e^{pt}Q_u(p)\,\tilde{\mathbf{G}}(\mathbf{r}, K_1(p), K_2(p)),\quad \gamma = (-i\infty; +i\infty).$$

If there exists proportionality between the mappings of the kernels M(p) and L(p), so that $K_1(p)/K_2(p) = \text{const}$, this integral can be neglected, using the Efros theorem of generalized convolution [1, 5]:

$$\mathbf{u}(\mathbf{r}, t) = \int_0^t dt'\,Q_u(t')\int_0^\infty d\zeta\,\mathbf{G}(\mathbf{r}, \zeta)\,I(t-t', c_s\zeta), \tag{3}$$

where $\mathbf{G}(\mathbf{r}, t) = \frac{1}{2\pi i}\int_\gamma dp\,e^{pt}\mathbf{G}\left(\mathbf{r}, \frac{p}{c_l}, \frac{p}{c_s}\right)$ is the space–time representation of the Green's function of the displacement field in an elastic medium, and

$$I(t, x) = \frac{1}{2\pi i}\int_\gamma dp\,\exp\{pt - xK_2(p)\} \tag{4}$$

is a factor accounting for the dispersion-dissipative medium properties.

Rayleigh Waves in an Elastic Medium. For an elastic medium the Green's function of the problem under consideration is well known [3, 6]. We are interested only in the surface displacement. In this case the Green's function of the surface displacement G can be represented in the form

$$\mathbf{G}(x, h, t) = \frac{1}{2\pi i}\int_{\gamma} dp\, e^{pt}\frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, e^{ikx}\mathbf{G}(k, h, p). \tag{5}$$

Here

$$\mathbf{G}(k, h, p)=\{G_z(k, h\, p);\, G_x(k, h, p)\};\quad G_z(k, h, p)=\frac{1}{4\pi}\frac{\frac{p^2}{c_s^2}\left(\frac{p^2}{c_s^2}+2k^2\right)}{D\left(\frac{p^2}{c_l^2}, \frac{p^2}{c_s^2}, k^2\right)}\times$$

$$\times\exp\left\{-h\sqrt{k^2+\frac{p^2}{c_l^2}}\right\};\; G_x(k, h, p)=\frac{1}{4\pi}\frac{2k\frac{p^2}{c_s^2}\sqrt{k^2+\frac{p^2}{c_s^2}}}{D\left(\frac{p^2}{c_l^2}, \frac{p^2}{c_s^2}, k^2\right)}\exp\left\{-h\sqrt{k^2+\frac{p^2}{c_l^2}}\right\};$$

$$D\left(\frac{p^2}{c_l^2}, \frac{p^2}{c_s^2}, k^2\right)=\left(\frac{p^2}{c_s^2}+2k^2\right)^2-4k^2\sqrt{k^2+\frac{p^2}{c_l^2}}\sqrt{k^2+\frac{p^2}{c_s^2}}.$$

Being interested only in that part of the surface displacement which is due to a Rayleigh wave and determined by the roots of the dispersion equation $D(p^2/c_l^2, p^2/c_s^2, k^2)=0$, we take into account that this equation has two roots: $k = \pm ip/c_R$ (c_R is the propagation velocity of the Rayleigh wave, $c_R < c_s$). The contribution to (5) due to poles corresponding to these roots leads to the following representation of the part of the surface displacement $\mathbf{G}^R$ corresponding to a Rayleigh wave:

$$G_z^R(x, h, t) = A_z\frac{\partial}{\partial x}\left(\frac{h_1}{(t-x/c_R)^2+h_1^2}+\frac{h_1}{(t+x/c_R)^2+h_1^2}\right), \tag{6}$$

where

$$G_x^R(x, h, t) = A_x\frac{\partial}{\partial x}\left(\frac{(t-x/c_R)}{(t-x/c_R)^2+h_1^2}-\frac{(t+x/c_R)}{(t+x/c_R)^2+h_1^2}\right);$$

$$A_z = -\frac{1}{8\pi^2}\frac{c_R^2}{c_s^2}\frac{1}{D^R}\left[\frac{2}{c_R^2}-\frac{1}{c_s^2}\right];\quad A_x = \frac{1}{8\pi^2}\frac{c_R^2}{c_s^2}\frac{1}{D^R}\left[\frac{2}{c_R}\left(\frac{1}{c_R^2}-\frac{1}{c_s^2}\right)^{1/2}\right];$$

$$h_1 = h\sqrt{\frac{1}{c_R^2}-\frac{1}{c_l^2}};\quad D^R=\left(\frac{\partial}{\partial k^2}D\left(\frac{p^2}{c_l^2}, \frac{p^2}{c_s^2}, k^2\right)\right)\Bigg|_{p^2=1,\; k^2=-1/c_R^2}.$$

Expressions (6) are not causal, while at the same time the full wave field determining (5) is causal and propagates with velocity c_l. This fact is explained in [7], and is related to the fact that in the region of distances $R = \sqrt{x^2 + h^2} > c_l t$ the contribution of Rayleigh poles to the total displacement field is completely compensated by the contributions of other singularities of the integrand expression in (5). To explain the causality of the expressions we multiply them by the cutoff factor $\theta(t-(\sqrt{x^2+h^2})/c_l)$, where $\theta(t)=\begin{cases}1, & t>0,\\ 0, & t\leqslant 0.\end{cases}$ This refinement makes it possible to analyze simply the formation process of the Rayleigh wave. In particular, the Rayleigh wave can be assumed to be formed if the following condition is satisfied $x/c_R - mh_1 \geqslant (\sqrt{x^2+h^2})/c_l$, implying that the interval between the wave front and its center, moving with velocity c_R is not less than m characteristic time scales h_1 of the wave. This condition leads to the form

$$x \geqslant \frac{c_R h}{\sqrt{c_l^2-c_R^2}}\left[\sqrt{1+m^2}+m\frac{c_l}{c_R}\right], \tag{7}$$

from which one obtains at m = 0 the well-known Nakano condition [3], which can be interpreted as the distance at which half of the Rayleigh wave is formed. This estimate is necessary for restricting the region where the results obtained below are valid for the formation of Rayleigh waves determined by expressions (6). Expressions (6) show that in an ideal elastic medium the characteristic period of a Rayleigh wave is determined by the source depth h. In particular, if the source is on the surface, i.e., h = 0, the vertical displacement component is

$$G_z^R(x, h=0, t) = \pi A_z\frac{\partial}{\partial x}\left(\delta\left(t-\frac{x}{c_R}\right)+\delta\left(t+\frac{x}{c_R}\right)\right), \tag{8}$$

so that its period and formation length vanish. At distances exceeding the length of formation, Rayleigh waves moving from right to left are well separated, so that in the following one may consider only the wave propagating from the right, determined by the expressions

$$G_z^R(x, h, t) = A_z \frac{\partial}{\partial x} \frac{h_1}{(t - x/c_R)^2 + h_1^2},$$
$$G_x^R(x, h, t) = A_x \frac{\partial}{\partial x} \frac{(t - x/c_R)}{(t - x/c_R)^2 + h_1^2}. \tag{9}$$

<u>Rayleigh Waves in a Dispersion-Dissipative Medium.</u> Expression (3) related the Rayleigh wave in an inelastic medium with the Rayleigh wave in an elastic medium in terms of its convolution with kernel I(t, x); therefore, it is required to know the explicit form of the factor I(t, x). It is shown in the Appendix that it has the asymptotic representation

$$I(t, x) = l\sqrt{\frac{f}{x}} \exp\left\{-\frac{f}{x}\left(t - \frac{x}{gc_{s\infty}}\right)^2\right\}, \tag{10}$$

valid in the region of values $x \gg c_{s\infty}\tau$, where $c_{s\infty}$ is the sound velocity of transverse waves in the dispersion-dissipative medium in the high-frequency limit $\omega \to \infty$, and τ is the characteristic relaxation time of the medium.

Following substitution into (3) of expressions (9) and (10) it can be transformed to the form

$$\begin{Bmatrix} \tilde{G}_z^R \\ \tilde{G}_x^R \end{Bmatrix} = \begin{Bmatrix} A_z \\ A_x \end{Bmatrix} \frac{\partial}{\partial x} \int_0^\infty d\zeta\, l \sqrt{\frac{f}{\zeta c_{s\infty}}} \exp\left\{-\frac{f}{\zeta c_{s\infty}}\left(t - \frac{\zeta}{g}\right)^2\right\} * \frac{1}{2}\left[\left(\zeta - \frac{x}{c_R} - ih_1\right)^{-1} \mp \left(\zeta - \frac{x}{c_R} + ih_1\right)^{-1}\right].$$

Calculating the integral [8], we obtain

$$\begin{Bmatrix} \tilde{G}_z^R \\ \tilde{G}_x^R \end{Bmatrix} = \begin{Bmatrix} A_z \\ A_x \end{Bmatrix} \frac{\partial}{\partial x} \frac{\pi l}{2} \sqrt{\frac{f}{c_{s\infty}}} \left[\left(\frac{x}{c_R} + ih_1\right) \exp\left\{-\frac{f}{c_{s\infty}} \frac{\left(t - \frac{x}{gc_R} + i\frac{h_1}{g}\right)^2}{(x/c_R + ih_1)}\right\} * \right.$$
$$\left. * \operatorname{erf} c\left\{i\sqrt{\frac{f}{c_{s\infty}(x/c_R + ih_1)}}\left(t - \frac{x/c_R + ih_1}{g}\right)\right\} \pm \text{c.c.}\right] \tag{11}$$
$$\left(\operatorname{erf} c(z) = 1 - \operatorname{erf}(z) = 1 - \frac{2}{\sqrt{\pi}} \int_0^z dt \exp(-t^2)\right).$$

If the condition $x \ll \frac{fc_R}{c_{s\infty}}\left[\left(t - \frac{x}{gc_R}\right)^2 + \frac{h_1^2}{g^2}\right]$ is satisfied, one can use the asymptotic representation at large error of the probability integral $\operatorname{erf}(z) = 1 - \frac{1}{\sqrt{\pi}z}\exp(-z^2)$, leading to (9) accurately within $l\sqrt{\pi}$. Thus, when this condition is satisfied the factor I(t, x) is effectively a delta-function with respect to G^R. If the condition $x \gg \frac{fc_R}{c_{s\infty}}\left[\left(t - \frac{x}{gc_R}\right)^2 + \frac{h_1^2}{g^2}\right]$, is satisfied, the function erf c(z) in (11) can be expanded for small values of the argument $\operatorname{erf} c(z) = 1 - \frac{2}{\sqrt{\pi}}z + \ldots$. We note that to calculate $\tilde{G}_z^R$ the first expansion term is sufficient, while for $\tilde{G}_x^R$ one also needs to account for the second term. Further, according to (7), neglecting whenever possible $h_1 c_R$ in comparison with x, we find the final expression

$$\begin{Bmatrix} \tilde{G}_z^R \\ \tilde{G}_x^R \end{Bmatrix} = \frac{\pi l}{2} \sqrt{\frac{f}{c_{s\infty}}} \frac{\partial}{\partial x} \begin{Bmatrix} A_z \frac{2}{\sqrt{x}} \\ A_x \frac{2}{\sqrt{\pi}} \frac{c_{s\infty}}{f} \frac{\partial}{\partial t} \end{Bmatrix} \exp\left\{-\frac{fc_R}{xc_{s\infty}}\left(t - \frac{x}{gc_R}\right)^2\right\}, \tag{12}$$

from which it follows that at distances $x \gg g^2 c_{s\infty} c_R / f$ the components of the vector $\tilde{G}^R$ have the quite simple form:

$$\tilde{G}_z^R = \sqrt{\pi} l \sqrt{\frac{f}{c_{s\infty}}} A_z \frac{2fc_R}{x^{3/2} c_{s\infty}}\left(t - \frac{x}{gc_R}\right) \exp\left\{-\frac{fc_R}{xc_{s\infty}}\left(t - \frac{x}{gc_R}\right)^2\right\},$$
$$\tilde{G}_x^R = \frac{l}{\sqrt{\pi}} \sqrt{\frac{f}{c_{s\infty}}} A_x \frac{2}{gc_R x}\left(1 - \frac{2fc_R}{gxc_{s\infty}}\left(t - \frac{x}{gc_R}\right)^2\right] \exp\left\{-\frac{fc_R}{xc_{s\infty}}\left(t - \frac{x}{gc_R}\right)^2\right\}. \tag{13}$$

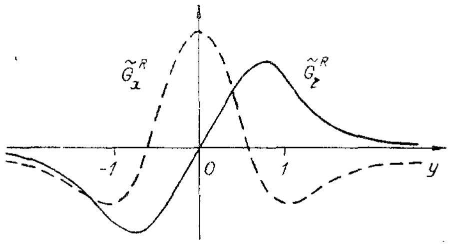

Fig. 1

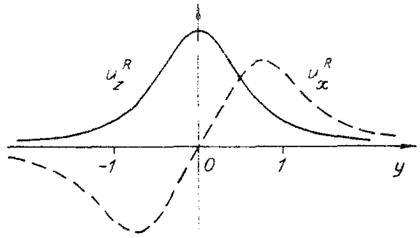

Fig. 2

The plots of the functions $\tilde{G}_z^R$ and $\tilde{G}_x^R$ are shown in Fig. 1 as a function of the parameter $y = -\sqrt{\frac{fc_R}{xc_{s\infty}}}\left(t - \frac{x}{gc_R}\right)$, corresponding to a transition to an associated coordinate system moving with velocity gc_R. It also follows from (13) that the amplitude $\tilde{G}^R$ decays with distance according to the law x^{-1}, while its characteristic width increases proportionately to $x^{1/2}$.

An expression for the vertical component of $\tilde{G}_z^R$ can be obtained more simply, taking into account that for $x \gg \frac{c_R}{c_{s\infty}}\frac{h_1^2}{g^2}f$ the source can be considered as a surface source with the Green's function (8), leading directly to the expression $\tilde{G}_z^R = \pi A_z \frac{\partial}{\partial x}\left[I\left(t, x\frac{c_{s\infty}}{c_R}\right) + I\left(t, -x\frac{c_{s\infty}}{c_R}\right)\right]$, whence follows the representation (12), with account of (10), for the wave propagating to the right.

Asymptotic Velocities and Displacements in a Rayleigh Wave. The surface displacement in a Rayleigh wave is expressed in terms of its Green's function $\tilde{\mathbf{G}}^R$ according to (3), while its velocity is expressed in the form

$$\mathbf{v}^R(\mathbf{r}, t) = \int_0^t dt' Q_v(t')\, \tilde{\mathbf{G}}^R(\mathbf{r}, t - t'), \tag{14}$$

where $Q_v(t)$ is the source function for the velocity: $Q_v(t) = \frac{d}{dt}Q_u(t)$. The line source function $Q_u(t)$ is related to the displacement $u(r_0, t) = U_0F(t)$, assigned at the surface of the corresponding cylindrical source of radius r_0 by the limiting transition $r_0 \to 0$, $U_0 \to \infty$, $r_0U_0 = \text{const}$, so that $Q_u(t) = 2\pi r_0 U_0 F(t)$.

Consider the case often encountered practically, when $Q_v(t)$ has the shape of a single pulse with characteristic radiation time T_0. For $x \gg \frac{c_R}{c_{s\infty}} f\left(T_0^2 + \frac{h_1^2}{g^2}\right)$, then, the function in expression (14) can be placed outside the integral sign at the maximum point of the function $Q_v(t)$, which, for simplicity, we assume is equal to zero:

$$\mathbf{v}^R(x, t) = \tilde{\mathbf{G}}^R(x, t)\int_0^t dt' Q_v(t'). \tag{15}$$

Expression (15) shows that in the region of large distances

$$x \gg \max\left\{\frac{c_R h}{\sqrt{c_l^2 - c_R^2}};\ c_{s\infty}\tau;\ \frac{c_R}{c_{s\infty}} f\left(T_0^2 + \frac{h_1^2}{g^2}\right)\right\}, \tag{16}$$

which with account of all restrictions used are determined by the dissipative medium properties and by source parameters such as the radiation time and the layer depth, while the profile of the mass velocity in the Rayleigh wave has a universal structure for the given geometry of the problem (see Fig. 1), independently of the specific shape of $Q_v(t)$. While the amplitude of the velocity decreases with distance by the law x^{-1}, the characteristic pulse width equals $\sqrt{c_{s\infty}x/c_R f}$.

We note that under the condition $x \gg g^2 c_{s\infty} c_R / f$ the approximate relation holds $\frac{\partial}{\partial x} I(t, x) = \frac{-1}{gc_R}\frac{\partial}{\partial t} I(t, x)$, using which, the expressions for the velocity and displacement are represented in the form

$$\begin{Bmatrix}\mathbf{u}^R(x, t)\\ \mathbf{v}^R(x, t)\end{Bmatrix} = \mathbf{B}\int_0^\infty dt' \begin{Bmatrix} Q_v(t') \\ \dot{Q}_v(t')\end{Bmatrix} \exp\left\{-\frac{fc_R}{xc_{s\infty}}\left(t - t' - \frac{x}{gc_R}\right)^2\right\},$$

$$\mathbf{B} = \pi l \sqrt{\frac{f}{c_{s\infty}}} \begin{Bmatrix} \frac{A_z}{gc_R} x^{-1/2} \\ -\frac{A_x c_{s\infty}}{\sqrt{\pi f}} \frac{\partial}{\partial x} \end{Bmatrix}. \tag{17}$$

It follows from (17) that at large distances (16) the profiles of the displacement components have the shape shown in Fig. 2, while their amplitude decays with distance according to the law $x^{-1/2}$. It also follows from (17) that if for Q_V the characteristic time of signal growth T_1 [it determines the width of $Q_V(t)$] is small with respect to the radiation time T_0, then in the region of intermediate distances $\max\left\{\frac{c_R h}{\sqrt{c_l^2 - c_R^2}};\ c_{s\infty}\tau;\ \frac{c_R}{c_{s\infty}} f\left(T_1^2 + \frac{h_1^2}{g^2}\right)\right\} \ll x \ll \frac{c_R}{c_{s\infty}} f T_0^2$ the velocity and displacement components are given by the expression

$$\begin{Bmatrix} v_z^R(x, t) \\ v_x^R(x, t) \end{Bmatrix} = \pi l \sqrt{\frac{f}{c_{s\infty}}} Q_v(t) \begin{Bmatrix} \frac{A_z}{gc_R} x^{-1/2} \\ -\frac{A_x}{\sqrt{\pi}} \frac{c_{s\infty}}{fx} 2\left(t - \frac{x}{gc_R}\right) \end{Bmatrix} * \exp\left\{-\frac{f}{x}\frac{c_R}{c_{s\infty}}\left(t - \frac{x}{gc_R}\right)^2\right\},$$

$$\begin{Bmatrix} u_z^R(x, t) \\ u_x^R(x, t) \end{Bmatrix} = \pi l \sqrt{\frac{f}{c_{s\infty}}} Q_v(t) \begin{Bmatrix} \frac{A_z}{gc_R} \operatorname{erf}\left\{-\sqrt{\frac{fc_R}{xc_{s\infty}}}\left(t - \frac{x}{gc_R}\right)\right\} \\ -\frac{A_x}{\sqrt{\pi}} \frac{c_{s\infty}}{f} \exp\left\{-\frac{fc_R}{xc_{s\infty}}\left(t - \frac{x}{gc_R}\right)^2\right\} \end{Bmatrix}.$$

Here $Q_V(t)$ gives only the amplitude, and therefore, in particular, the profiles of the velocity components have the shape shown in Fig. 2, while their amplitude decays according to the law $x^{-1/2}$.

Estimates of the Region of Asymptotic Behavior of a Rayleigh Wave. Relationship (16) determines the region of large distances satisfying the following conditions: the Rayleigh wave can be assumed to be formed by (7); the dissipative factor I(t, x) has the shape (10) ($x \gg c_{s\to}\tau$); the profile of the Rayleigh wave under the action of dissipative medium properties undergoes the universal shape (15) $\left(x \gg f\frac{c_R}{c_{s\infty}}\left(T_0^2 + \frac{h_1^2}{g^2}\right)\right)$.

We provide several estimates, showing the role of various parameters of the source and of the medium. In seismology one encounters sources with $T_0 \sim 0.1$ sec. Assuming that this source exists in the medium at a moist carbonate loam, for which by the data of [9] ($\tau \sim 10^{-5}$ sec, $c_S \sim 0.26$ km/sec, $a = c_{S0}^2/c_{S\infty}^2 \sim 0.92$), we find, using expression (A.6) for the generalized medium relaxation (a standard body), that $f \sim 10^7$ km/sec^2. In this case condition (16) reduces to $x \gg 2\cdot10^4 h^2 + 2\cdot10^2$, where x and h are expressed in kilometers. It is seen that the region of asymptotic behavior of the Rayleigh wave is determined at h > 100 m by the source depth h, while for h < 100 m it is determined by the radiation time T_0. In the latter case the asymptotic behavior starts at $x \gtrsim 200$ km. In microelectronics $LiNbO_3$ is widely used. For it, from the data of [10] ($\tau \sim 10^{-11}$ sec is the thermal relaxation time, $c_S \sim 4$ km/sec, $a \sim 0.94$) and by Eq. (A.6) we find $f \sim 10^{13}$ km/sec. Hence, for a surface source with $T_0 \sim 5\cdot10^9$ sec the asymptotic behavior of a Rayleigh wave will be observed at distances $x \gg 0.4$ mm, which is comparable with the sizes of microelectronic devices, and therefore must be accounted for.

The examples provided show that for correct interpretation of the information transported by a nonmonochromatic Rayleigh wave in a dispersion-dissipative medium it is important to know whether or not the Rayleigh wave tends to its asymptote, which is determined by the source and medium parameters.

Thus, using an approach based on representing the solution in the form (3), when the factors corresponding to the geometry of the problem and to the dispersion-dissipative properties of the medium, we manage to analyze consistently the propagation of a Rayleigh wave in a nonideal medium and to answer the questions stated at the beginning of this paper.

Appendix: Properties of the Factor I(t, x). It follows from the representation (4) that the properties of I(t, x) are determined by the shape of the dependence $K_2(p)$. We introduce the notation $K_2^2(p) = \rho_0 p^2/M(p) \equiv p^2/\varkappa(p)$.

All various dispersion-dissipative properties of geomedia, determined by the relaxation kernel M(p) or $\varkappa(p)$, can be described phenomenologically in terms of the spectrum of the exponential relaxation time. The general form of the relaxation kernel is in this case

$$\varkappa(t) = c_\infty^2 \delta(t) - \Lambda \int_0^\infty g(\tau) \exp\left\{-\frac{t}{\tau}\right\} d\tau, \tag{A.1}$$

where the first term corresponds to the elastic part of the relaxation kernel, and the second corresponds to the inelastic part. The function $g(\tau)$ is the relaxation time spectrum with the properties: $\int_0^\infty g(\tau)\,d\tau = 1$, $g(\tau) \geqslant 0$, $g(\tau = 0) = 0$, the latter following from the fact that the contribution of $\tau = 0$, corresponding to ideal elasticity, was already extracted; Λ is a normalization constant. The relaxation time spectrum can be found experimentally or by physical considerations. It follows from (A.1) that

$$\varkappa(p) = c_\infty^2 - \Lambda \int_0^\infty \frac{g(\tau)}{p + 1/\tau}\, d\tau. \tag{A.2}$$

We note several properties of the mapping $\varkappa(p)$, independent of the specific shape of the spectrum $g(\tau)$. It follows from the causality principle that $K_2(p)$ is an analytic function in the region $\mathrm{Re}\, p \geq 0$, and it follows from (A.2) that $\varkappa(p)$ increases monotonically for real $p > 0$. This leads to the conclusion that $I(t, x) = 0$ for $x > c_\infty t$. We note that the factor $I(t, x)$ is essentially the Green's function of the one-dimensional planar problem for an unbounded dispersion-dissipative medium, so that one can talk about a perturbation propagation rate. The minimum velocity is found from the expression $c_0^2 = \lim_{p\to 0} \varkappa(p) = c_\infty^2 - \Lambda \int_0^\infty g(\tau)\,\tau d\tau$.

For $p \to \infty$ we obtain the maximum possible perturbation propagation rate $c_\infty^2 = \lim_{p\to\infty} \varkappa(p)$. Near the front $x = c_\infty t$ the structure of $I(t, x)$ is determined by the expansion (for $p \to \infty$) $K_2(p) = p/c_\infty + b_0 - b_1/p + \ldots$, and is [11]

$$I(t, x) = \exp\{-b_0 x\}\left[\delta\left(t - \frac{x}{c_\infty}\right) - \left(\frac{b_1 x}{t - x/c_\infty}\right)^{1/2} * I_1\left(2\sqrt{b_1 x (t - x/c_\infty)}\right) \theta(t - x/c_\infty)\right], \tag{A.3}$$

where $I_1(z)$ is the Bessel function of an imaginary argument. The expansion (for $p \to 0$) $K_2(p) = p/c_0 - a_0 p^2 + \ldots$ determines the structure of $I(t, x)$ near the point moving with velocity c_0:

$$I(t, x) = \sqrt{\frac{2}{\pi a_0 x}} \exp\left\{-\frac{(t - x/c_0)^2}{2 a_0 x}\right\}. \tag{A.4}$$

It follows from (A.3) and (A.4) that near the front $x = c_\infty t$ there is exponential damping with distance of the elastic indicator, following which propagates part of the pulse, decaying more weakly and described asymptotically by expression (A.4). To determine the behavior of $I(t, x)$ behind the front it was calculated by the steepest descent method for several specific relaxation time spectra $g(\tau)$ [12]. Thus, $g(\tau) = \delta(\tau - \tau_1)$ corresponds to the generalized relaxation medium (a standard body). A Voight medium is obtained from the following limiting transition $\tau_1 \to 0$, $c_\infty \to \infty$, $\tau_1 c_\infty^2 / c_0^2 \to \tau'$. A Gurevich medium was also considered. For the first two cases we showed uniqueness of the saddle point, while for a Gurevich medium a similar result was obtained for real p. In all cases the saddle point is located on the real axis, and the saddle contour passes perpendicularly to the latter. The validity of the steepest descent method is determined by the condition $x/(c_\infty \tau) \gg 1$ (τ is characteristic of the selected relaxation time model). The same condition makes it possible to represent $I(t, x)$ in the form

$$I(t, x) = l \sqrt{\frac{f}{x}} \exp\left\{-\frac{f}{x}\left(t - \frac{x}{g c_\infty}\right)^2\right\}, \tag{A.5}$$

where $l = 1/(2\sqrt{\pi})$, and the parameters f and g are the following:

for a Voight medium $f = 2c_0/\tau'$, $g = 1$;

for generalized medium relaxation

$$f = 2\frac{c_\infty}{\tau_1}\frac{a^{3/2}}{1 - a},\ g = a^{1/2},\ a = \frac{c_0^2}{c_\infty^2}; \tag{A.6}$$

for a Gurevich medium

$$f = 2\frac{c_\infty}{\tau}\frac{1}{A}\frac{a'}{1-a'}(1 - A\ln a')^{1/2},\ g = (1 - A\ln a')^{-1/2}.$$

Here A is the ratio of the shear elastic modulus to the relaxation one, and a' is the ratio of minimum to maximum relaxation times.

Comparison of (A.5) and (A.4) shows that at distances $x \gg c_\infty\tau$ the maximum of I(t, x) is displaced with velocity c_0, and is determined by the expansion of $K_2(p)$ near the point p = 0. We note that for $\tau \to 0$ there exists for the representation (A.5) a limiting transition to the case of an ideal elastic medium $I(t, x) = \delta(t - x/c_\infty)$, since $f \sim \tau^{-1}$.

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STUDY OF ELASTOPLASTIC DEFORMATION FOR CYLINDRICAL SHELLS WITH AXIAL SHOCK LOADING

A. I. Abakumov, G. A. Kvaskov,
S. A. Novikov, V. A. Sinitsyn,
and A. A. Uchaev

UDC 620.178.7

There is considerable practical interest in studying the dynamic stability of cylindrical shells under the action of axial intense shock loads. A shell is assumed to be dynamically stable if its movement is not accompanied by buckling, i.e., it is constrained. The nature of loss of stability for a cylindrical shell is determined mainly by its relative thickness h/R (h is shell thickness, R is central surface radius). For relatively thin shells with $h/R < 1/100$ elastic buckling is normally considered when loss of stability occurs with formation of rhombic hollows, and shell deflection as a result of sudden popping. With an increase in relative shell thickness plastic buckling is observed during its axial compression. Plastic loss of stability is characterized by the fact that the shell may demonstrate marked resistance to buckling. In the initial stage of deformation with plastic buckling there is almost always axisymmetrical loss of stability in the form of an annular fold caused by the effect of boundary conditions at the shell edges. With further axial compression the shell continues to lose stability in axisymmetrical shape or it may change over to an asymmetrical form of loss of stability. It was shown by experiment in [1] that the form of loss

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 150-153, May-June, 1988. Original article submitted February 3, 1987.

0021-8944/88/2903-0450$12.50